ENTRAINMENT OF A VISCOPLASTIC FLUID BY A ROTATING DRUM

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A rotating drum partially immersed in a viscoplastic fluid is considered and the thickness of the film as well as the amount of fluid entrained by it are determined.

In many technological processes, various surfaces are coated with fluids. Typical examples are deposition of protective or decorative coatings on paper, cloth, and metal or manufacture of photographic materials and sound recording tapes. The equipment used for deposition of fluid coatings often includes rotating drums, cylinders, or rollers which drag the fluid out of the vat and feed it onto the substrate. It is then possible to determine the thickness of the deposited layer if the characteristics of fluid entrainment by a rotating surface have been established.

We consider a rotating drum (Fig. 1a) partially immersed in a Shvedov–Bingham viscoplastic fluid

$$\tau=\left(\operatorname{sign}\frac{\partial u}{\partial y}\right)\tau_0+\mu_p\frac{\partial u}{\partial y},\ |\tau|>\tau_0,\quad \frac{\partial u}{\partial y}=0,\ |\tau|\leqslant\tau_0.$$

We will calculate the thickness of the film h and the amount of fluid (flow rate) q entrained by a unit of drum surface area in a unit of time. In the steady state the fluid flow rate q remains uniform around the drum circumference, i.e., independent of the angle φ, unlike the film thickness h, which is generally a function of the angle φ. The drum radius is assumed to be much larger than the film thickness h. Then the segment of the drum surface emerging from the fluid can, within an acceptable accuracy, be approximated as a plane surface emerging from the fluid at some angle φ_0 [1]. This angle φ_0 will be called the angle of drum immersion in the fluid. The pulling of a plane surface obliquely out of a viscoplastic fluid (Fig. 1b) has been analyzed in an earlier study [2].

Following that earlier procedure [2], we write the equations of motion for the fluid film within the entrainment region as

$$\frac{\partial\tau}{\partial y}-\frac{\partial p}{\partial x}-\rho g\sin\varphi_0=0,\quad -\frac{\partial p}{\partial y}=\rho g\cos\varphi_0$$

with the boundary conditions

$$u=U\ \text{at}\ y=0,\ \tau=0\ \text{at}\ y=h,\ p=p_0-\sigma\frac{d^2h}{dx^2}\ \text{at}\ y=h.$$

The equation describing the shape of the fluid surface within the region of the static meniscus is

$$\frac{d^2h}{dx^2}\left[1+\left(\frac{dh}{dx}\right)^2\right]^{-3/2}=\frac{\rho g}{\sigma}x(1-\cos\varphi_0).$$

From the condition of collocation of the solutions in the entrainment region and in the region of the static meniscus, this condition being that the surface curvature remain continuous at the boundary between both regions, one can obtain an expression for calculating the film thickness in the case of an oblique plane surface pulling out of the fluid. In the case of a Shvedov–Bingham viscoplastic fluid we obtain for the dimensionless film thickness [2] the expression

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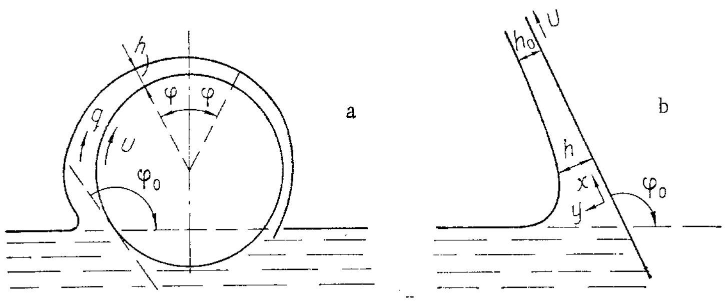

Fig. 1. Schematic diagram depicting the entrainment of a fluid by (a) a rotating drum and (b) an oblique plate.

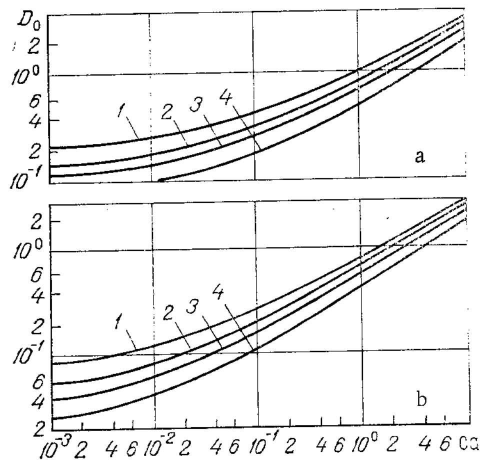

Fig. 2. Dependence of the film thickness D_0 on Ca with B = 0.3 (a) and B = 0.15 (b): 1) φ_0 = 60°; 2) φ_0 = 90°; 3) φ_0 = 120°; 4) φ_0 = 150°.

$$D_0 = D(B,\ \varphi_0) + D(\mathrm{Ca},\ \varphi_0). \tag{1}$$

Here

$$D(B,\ \varphi_0) = B - \frac{1}{9.16}\left\{\frac{1-\cos\varphi_0}{\sin\varphi_0}\right)^{3/2}\left\{\left[1 + 18.32B\left(\frac{\sin\varphi_0}{1-\cos\varphi_0}\right)^{3/2}\right]^{1/2} - 1\right\}, \tag{2}$$

$D(\mathrm{Ca},\ \varphi_0)$ is determined from the relation

$$D(\mathrm{Ca},\ \varphi_0) = 0.944\left(\frac{\sin\varphi_0}{1-\cos\varphi_0}\right)^{1/2}[\mathrm{Ca} - D^2(\mathrm{Ca},\ \varphi_0)]^{2/3}, \tag{3}$$

with the dimensionless dependent and independent variables D, B, and Ca defined as

$$D = h\left(\frac{\rho g}{\sigma}\sin\varphi_0\right)^{1/2},\quad B = \tau_0/(\rho g\sigma\sin\varphi_0)^{1/2},\ \mathrm{Ca} = \frac{\mu_p U}{\sigma}. \tag{4}$$

The dependence of the film thickness D_0 on the entrainment velocity Ca and on the inclination angle φ_0 and the plasticity parameter B is shown in Fig. 2.

We will now determine the amount of fluid entrained by a rotating cylindrical drum. It is to be noted, first of all, that there exists a limiting thickness for a film of viscoplastic fluid beyond which it will cease to run down under the forces of gravity but will, instead, move as one entity together with the pulling surface. This limiting film thickness at the point where the drum emerges from the fluid, i.e., at the immersion angle φ_0, can be determined from the balance of gravity and friction forces, viz.

$$h_1 = \frac{\tau_0}{\rho g \sin \varphi_0}$$

or in dimensionless form (4)

$$D_1 = B.$$

When the thickness of the entrained film is $D_0 \leqslant B$, therefore, then the amount of entrained viscoplastic fluid will be

$$q = Uh_0$$

or in dimensionless form

$$Q = \mathrm{Ca}\, D_0. \tag{5}$$

When the inequality $D_0 > B$ holds true, then the film of viscoplastic fluid will run down under the forces of gravity. For calculating the amount of entrained fluid, one must then use the relation [2]

$$q = Uh_0 - \frac{\rho g \sin \varphi_0}{3\mu_p} h_0^3 + \frac{\tau_0}{2\mu_p} h_0^2 - \frac{\tau_0^3}{6\mu_p (\rho g \sin \varphi_0)^2},$$

which in dimensionless form becomes

$$Q = \mathrm{Ca}\, D_0 - \frac{D_0^3}{3}\left(1 - \frac{3}{2}\frac{B}{D_0} + \frac{1}{2}\frac{B^3}{D_0^3}\right). \tag{6}$$

We note that with $D_0 = B$ expression (6) reduces to expression (5), which is valid for $D_0 \leqslant B$.

We will next determine the film thickness around the circumference of a rotating drum. We use the continuity equation in the integral form, which expresses a constant rate of fluid flow. Accordingly,

$$Q = \mathrm{Ca}\, D (\mp) \frac{D^3}{3} \frac{\sin \varphi}{\sin \varphi_0}\left[1 - \frac{3B}{2D}\frac{\sin \varphi_0}{\sin \varphi} + \frac{B^3}{2D^3}\left(\frac{\sin \varphi_0}{\sin \varphi}\right)^3\right],$$

$$0 \leqslant \varphi \leqslant \varphi_0. \tag{7}$$

Angles with the "–" sign are read counterclockwise, angles with the "+" sign are read clockwise (Fig. 1). Equation (7) is valid for film thicknesses

$$D \geqslant B \frac{\sin \varphi_0}{\sin \varphi}.$$

Here expression $B \sin \varphi_0 / \sin \varphi$ determines the magnitude of the limiting thickness of a viscoplastic film at some angle φ $(0 \leqslant \varphi \leqslant \varphi_0)$. The limiting film thickness is minimum and equal to $B \sin \varphi_0$ when $\varphi = 90°$, it increases boundlessly as angle becomes smaller or larger than 90°. When the film thickness around the drum circumference is smaller than limiting, i.e., when

$$D_0 \leqslant B \frac{\sin \varphi_0}{\sin \varphi} \tag{8}$$

at any point $0 \leqslant \varphi \leqslant \varphi_0$, then the film thickness is uniform and equal to D_0 around the drum circumference. For $\varphi_0 \leqslant 90°$ condition (8) changes to $D_0 \leqslant B$, while for $\varphi_0 > 90°$ we obtain $D_0 \leqslant B \sin \varphi_0$. Expressions (1)-(4) indicate that, with φ_0 and B given, D_0 is a function of the dimensionless velocity Ca only. Condition (8) implies the existence of a certain critical dimensionless velocity Ca* such that at any $\mathrm{Ca} \leqslant \mathrm{Ca}^*$ the film thickness does not depend on the angle φ. When

$$D_0 > B \frac{\sin \varphi_0}{\sin \varphi} \tag{9}$$

then D generally does not depend on the angle φ, but flow regions will necessarily exist where the film thickness remains uniform. In order to determine these regions, it is necessary to let

$$Q = \mathrm{Ca}\, D_\infty,$$

and

$$1-\frac{3}{2}\frac{B}{D_\infty}\frac{\sin\varphi_0}{\sin\varphi_\infty}+\frac{B^3}{2D_\infty^3}\left(\frac{\sin\varphi_0}{\sin\varphi_\infty}\right)^3=0$$

in Eq. (7). We then obtain

$$\sin\varphi_\infty=\frac{B\mathrm{Ca}}{Q}\sin\varphi_0. \tag{10}$$

Consequently, at angles $\sin\varphi < \sin\varphi_\infty$ the film thickness remains uniform in every case. We note that when $\varphi_0 \geqslant 90°$ and $B\sin\varphi_0 \leqslant D_0 \leqslant B$, the rate of fluid flow Q is determined from the relation (5), and expression (10) becomes

$$\sin\varphi_\infty=\frac{B}{D_0}\sin\varphi_0.$$

For determining the film thickness D around the circumference of a rotating drum, therefore, one must use expressions (1)-(4), calculate the thickness D_0 of the film entrained by a flat plate pulling out at the angle φ_0, then from either relation (5) or relation (6) find the rate of fluid flow Q, and finally from expressions (7)-(10) determine the film thickness as a function of the angle φ.

NOTATION

x, y, Cartesian coordinates; τ_0, yield point; μ_p, plastic viscosity; ρ, density; g, acceleration due to gravity; p, pressure; σ, surface tension; and U, linear velocity of the drum surface.

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COALESCENCE OF CONCENTRATED FINE-DISPERSE EMULSIONS DURING TURBULENT STIRRING

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The coalescence of droplets due to an average gradient of velocity fluctuations is considered and its dependence on the concentration of the dispersed phase is analyzed within the framework of the theory of locally isotropic turbulence.

The rate of separation of unstable emulsions which have formed during concurrent motion and mixing of mutually insoluble fluids determines the effectiveness of a great many technological processes in the chemical, petroleum, food, and various other industries. The enlargement of droplets of the dispersed phase during turbulent flow of unstable emulsions through pipelines makes it possible to increase the productivity of sedimentation and extraction equipment [1].

An analysis of the interaction between fine-disperse droplets during the flow of emulsions also facilitates the solution of problems pertaining to two-phase flow through pipelines. The true contents of each phase, the limits of existence of various stream structures, and transitions from one structure to another depend largely on the coalescence of droplets, and this dependence affects the hydrodynamics of a two-phase stream as well as the heat and mass transfer in it.

The object of this study will be to determine how the coalescence of droplets during turbulent flow of an emulsion through a pipeline depends on the concentration of the dis-

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